R1, del 1

1) a) $\underline{f'(x) = 6x^5 + 15x^4 + \frac{1}{x}}$

b) $g'(x) = 4x \cdot e^{2x-1} + 2x^2 \cdot e^{2x-1} \cdot 2$

$= \underline{4x(1+x)e^{2x-1}}$

c) $h'(x) = \frac{4 \cdot (x+2) - (4x-1) \cdot 1}{(x+2)^2} = \frac{4x+8-4x+1}{(x+2)^2}$

$= \underline{\underline{\frac{9}{(x+2)^2}}}$

2) a) $\ln(x^2) + \ln x = 12$

$\ln(x \cdot x) + \ln x = 12$

$\ln x + \ln x + \ln x = 12$

$3\ln x = 12$

$\ln x = 4$

$e^{\ln x} = e^4$

$\underline{x = e^4}$

b) $e^{2x} - e^x = 6$

$e^{2x} - e^x - 6 = 0$

$e^x = \frac{1 \pm \sqrt{(-1)^2 + 4 \cdot 1 \cdot 6}}{2}$

$= \frac{1 \pm 5}{2}$

$e^x = 3 \quad \vee \quad e^x = -2$ ikke løsn

$\ln e^x = \ln 3$

$\underline{x = \ln 3}$

3) $\vec{u} \cdot \vec{v} = -2$

$|\vec{u}| = 3 \quad |\vec{v}| = 2 \quad \Rightarrow \vec{u}^2 = 9 \quad \vec{v}^2 = 4$

$\vec{a} = 2\vec{u} + 3\vec{v} \quad \vec{b} = t \cdot \vec{u} + 5\vec{v}$

a) $\vec{a} \parallel \vec{b}$

$k \cdot \vec{a} = \vec{b}$

$k \cdot (2\vec{u} + 3\vec{v}) = t \cdot \vec{u} + 5\vec{v}$ $\qquad$ $k \cdot 2\cancel{\vec{u}} = t \cdot \cancel{\vec{u}}$

$k \cdot 3\cancel{\vec{v}} = 5\cancel{\vec{v}}$ $\qquad$ $\frac{5}{3} \cdot 2 = t$

$\underline{k = \frac{5}{3}}$ $\qquad$ $\underline{t = \frac{10}{3}}$

b) $\vec{a} \perp \vec{b}$

$(2\vec{u} + 3\vec{v}) \cdot (t \cdot \vec{u} + 5\vec{v}) = 0$

$2t\vec{u}^2 + 10\vec{u} \cdot \vec{v} + 3t\vec{u} \cdot \vec{v} + 15\vec{v}^2 = 0$

$2t \cdot 9 + 10 \cdot (-2) + 3t \cdot (-2) + 15 \cdot 4 = 0$

$18t - 20 - 6t + 60 = 0$

$12t = -40$

$\underline{\underline{t}} = \frac{-40}{12} = \underline{\frac{-10}{3}}$

4) $P(x) = 6x^3 - 5x^2 - 2x + 1$

a) $\underline{P(1)} = 6 \cdot 1^3 - 5 \cdot 1^2 - 2 \cdot 1 + 1 = 6 - 5 - 2 + 1 = \underline{0}$

$\Rightarrow \underline{P(x) : (x-1) \text{ går upp}}$

b) $(6x^3 - 5x^2 - 2x + 1) : (x-1) = 6x^2 + x - 1$

$$
\begin{array}{l}
\underline{6x^3 - 6x^2} \\
\quad x^2 - 2x \\
\quad \underline{x^2 - x} \\
\qquad -x + 1 \\
\qquad \underline{-x + 1} \\
\qquad\quad 0
\end{array}
$$

$6x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 6 \cdot (-1)}}{2 \cdot 6} = \dfrac{-1 \pm 5}{12}$

$\underline{x} = \dfrac{4}{12} = \underline{\dfrac{1}{3}} \vee \underline{x} = \dfrac{-6}{12} = \underline{-\dfrac{1}{2}}$

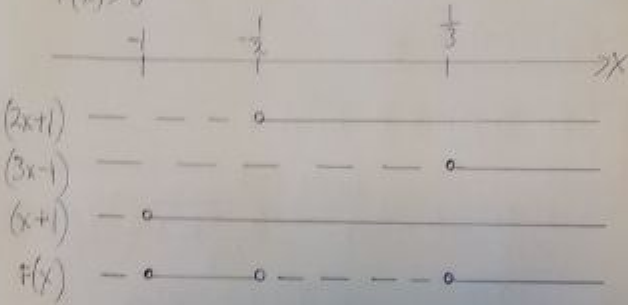
$\underline{P(x)} = 6(x-1)(x+\frac{1}{2})(x-\frac{1}{3}) = 2 \cdot 3(x-1)(x+\frac{1}{2})(x-\frac{1}{3})$

$= (x-1)\,2(x+\frac{1}{2})\,3(x-\frac{1}{3}) = \underline{(x-1)(2x+1)(3x-1)}$

$\longrightarrow$

c) $F(x) = \frac{f(x)}{x^2-1} = \frac{\cancel{(x-1)}(2x+1)(3x-1)}{\cancel{(x-1)}(x+1)} = \frac{(2x+1)(3x-1)}{x+1}$

$F(x) \geq 0$



$F(x) \geq 0$ for $x \in [-1, -\frac{1}{2}] \cup [\frac{1}{3} \rightarrow \rangle$

d) Betingelse: $x \neq -1$ (nevner kan ikke være 0!)

$\lim_{x \to 1} F(x) = \frac{(2 \cdot 1 + 1)(3 \cdot 1 - 1)}{1+1} = \frac{3 \cdot \cancel{2}}{\cancel{2}} = 3$

$\lim_{x \to -1} F(x)$ eksisterer ikke grunnet betingelsen $x \neq -1$

5) a) $\binom{8}{3} = \frac{8\cdot7\cdot\cancel{6}}{\cancel{3}\cdot\cancel{2}\cdot1} = \underline{\underline{56 \text{ mulige kombinasjoner}}}$

b) $\frac{\binom{3}{3}\cdot\binom{5}{1}}{\binom{8}{4}} = \frac{1\cdot5}{\frac{\cancel{8}\cdot7\cdot\cancel{6}^{2}\cdot5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1}} = \frac{5}{70} \quad (=P(X=3))$

$= \frac{1}{14}$

$\underline{\underline{\frac{1}{14} \text{ sannsynlig at han har riktig bok til alle fagene}}}$

c) X = ant riktige bøker

$\underline{\underline{P(X\geq2)}} = P(X=2)+P(X=3) = \frac{\binom{3}{2}\cdot\binom{5}{2}}{\binom{8}{4}} + \frac{5}{70}$

$= \frac{\frac{3\cdot\cancel{2}}{\cancel{2}\cdot1}\cdot\frac{5\cdot\cancel{4}^{2}}{\cancel{2}\cdot1}}{70} + \frac{5}{70} = \frac{3\cdot10}{70} + \frac{5}{70} = \frac{35}{70} = \underline{\underline{\frac{1}{2}}}$

$\underline{\underline{\frac{1}{2}\text{ (50\%) sannsynlig at han har minst to riktige bøker}}}$

a) $f(x) = 9 - x^2 \quad x \in \langle 0, 3 \rangle$

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x) = h$$

$$\underline{\underline{F(x)}} = \frac{(AB + DC) \cdot h}{2} = \frac{(6 + (x - (-x))) \cdot (9 - x^2)}{2}$$

$$= \frac{(6 + 2x)(9 - x^2)}{2} = \frac{54 - 6x^2 + 18x - 2x^3}{2}$$

$$= \underline{\underline{-x^3 - 3x^2 + 9x + 27}} \quad \underline{\underline{x \in \langle 0, 3 \rangle}}$$

b) $F'(x) = -3x^2 - 6x + 9 = 0 \quad | :(-3)$

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2} = \frac{-2 \pm 4}{2}$$

$$x = 1 \lor \cancel{x = -3}$$

$$\underline{\underline{F(1)}} = -1^3 - 3 \cdot 1^2 + 9 \cdot 1 + 27 = -1 - 3 + 9 + 27 = \underline{\underline{32}}$$

$$F''(x) = -6x - 6$$

$\underline{F''(1)} = -6 \cdot 1 - 6 = \underline{-12}$ (neg $\Rightarrow$ vi har et <u>topp-punkt!</u>)

7) $\angle u$ og $\angle D$ spenner over samme bue (AB) og er begge periferivinkler

$= \underline{\angle u = \angle D = 65^\circ}$

$\angle w$ er sentralvinkel som spenner over samme bue (AB) som periferivinkelen D

$= \underline{\angle w} = 2 \cdot \angle D = 2 \cdot 65^\circ = \underline{130^\circ}$

$\underline{\angle BEC} = 180^\circ - \angle CBE - \angle u = 180^\circ - 35^\circ - 65^\circ = \underline{80^\circ}$

$\underline{\angle v} = 180^\circ - \angle BEC = 180^\circ - 80^\circ = \underline{100^\circ}$

8) $A(-1,1)$ $C(7,5)$ linja $y = 2x+1$ gir $l: \begin{cases} x = t \\ y = 2t+1 \end{cases}$ $\vec{r}_l = [1,2]$

a) D ligger på $l \Rightarrow D(t, 2t+1)$ gir $\overrightarrow{AD} = [t-(-1), (2t+1)-1] = \underline{[t+1, 2t]}$

b) $\overrightarrow{CD} = [t-7, (2t+1)-5] = \underline{[t-7, 2t-4]}$

$|\overrightarrow{AD}| = |\overrightarrow{CD}|$

$(t+1)^2 + (2t)^2 = (t-7)^2 + (2t-4)^2$

$t^2 + 2t + 1 + 4t^2 = t^2 - 14t + 49 + 4t^2 - 16t + 16$

$2t + 14t + 16t = 49 + 16 - 1$

$32t = 64$

$\underline{t = 2}$

$\Rightarrow D(t, 2t+1) = D(2, 2\cdot 2+1) = \underline{D(2,5)}$

$\overrightarrow{AB} = \overrightarrow{DC}$ $B(a,b)$

$[a-(-1), b-1] = [5,0]$

$a+1 = 5$

$\underline{a = 4}$

$b-1 = 0$

$\underline{b = 1}$

$\Rightarrow B(a,b) = \underline{B(4,1)}$