

R2, Vår 20, del 2

1) $f(x) = x \sin x$

a) $f'(x) = \sin x + x \cdot \cos x$

b) $g(x) = \frac{\cos(x^2)}{x}$

$g'(x) = \frac{-\sin(x^2) \cdot 2x \cdot x - \cos(x^2) \cdot 1}{x^2} = -\frac{(2x^2 \sin(x^2) + \cos(x^2))}{x^2}$

2) a) $\int (x^2 + 3 + e^{2x}) dx = \frac{1}{3}x^3 + 3x + \frac{1}{2}e^{2x} + C$

b) $\int 6x \cdot \sin(x^2) dx$ • $x^2 = u$
 $2x dx = du$
 $dx = \frac{du}{2x}$

$\int 6x \cdot \sin u \cdot \frac{du}{2x} = -3 \cos u + C = -3 \cos(x^2) + C$

c) $\int_1^e x \cdot \ln x dx = \left[\ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]_1^e$
 $= \left[\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 \right]_1^e = \frac{1}{2} e^2 \ln e - \frac{1}{4} e^2 - \left(\frac{1}{2} \cdot 1^2 \ln 1 - \frac{1}{4} \cdot 1^2 \right)$
 $= \frac{1}{2} e^2 - \frac{1}{4} e^2 + \frac{1}{4} = \frac{1}{4} e^2 + \frac{1}{4} = \frac{e^2 + 1}{4}$

$$3) a_1 = 3 \quad S_5 = 55$$

$$a) S_n = \frac{n(a_1 + a_n)}{2} \quad S_5 = \frac{(a_1 + a_5) \cdot 5}{2}$$

$$55 = \frac{(3 + a_5) \cdot 5}{2} \Rightarrow 15 + 5a_5 = 110$$

$$5a_5 = 95 \Rightarrow \underline{a_5 = \frac{95}{5} = 19}$$

$$\underline{d = \frac{a_5 - a_1}{4} = \frac{19 - 3}{4} = \frac{16}{4} = 4}$$

$$a_n = a_1 + (n-1) \cdot d$$

$$\underline{a_{10} = 3 + (10-1) \cdot 4 = 3 + 9 \cdot 4 = 39}$$

$$\underline{\underline{S_{10} = \frac{10(3+39)}{2} = \frac{10 \cdot 42}{2} = \frac{420}{2} = 210}}$$

$$b) 7 + \frac{7}{2} + \frac{7}{4} + \dots, \quad \underline{\underline{k = \frac{1}{2} \Rightarrow |k| < 1}}$$

\Rightarrow konvergente

$$\underline{\underline{S = \frac{a_1}{1-k} = \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = 7 \cdot 2 = 14}}}$$

$$4) f(x) = 2 \sin(\pi x + \pi) - 1 \quad x \in \langle -1, 3 \rangle$$

$$a) \text{ Maks. verdi} = 2 \cdot 1 - 1 = \underline{1}$$

$$\sin(\pi x + \pi) = 1$$

$$\pi x + \pi = \frac{\pi}{2} + n \cdot 2\pi \quad | : \pi$$

$$x + 1 = \frac{1}{2} + 2n$$

$$x = -\frac{1}{2} + 2n$$

$$\underline{x = -\frac{1}{2} + 2 \cdot 0 = -\frac{1}{2}}$$

$$\underline{x = -\frac{1}{2} + 2 \cdot 1 = \frac{3}{2}}$$

$$\Rightarrow \text{Topplet } \underline{\left(-\frac{1}{2}, 1\right) \text{ og } \left(\frac{3}{2}, 1\right)}$$

$$\text{Min. verdi} = 2 \cdot (-1) - 1 = \underline{-3}$$

$$\sin(\pi x + \pi) = -1$$

$$\pi x + \pi = \frac{3\pi}{2} + n \cdot 2\pi \quad | : \pi$$

$$x + 1 = \frac{3}{2} + 2n$$

$$x = \frac{1}{2} + 2n$$

$$\underline{x = \frac{1}{2} + 2 \cdot 0 = \frac{1}{2}}$$

$$\underline{x = \frac{1}{2} + 2 \cdot 1 = \frac{5}{2}}$$

$$\Rightarrow \text{Bunnpunkt } \underline{\left(\frac{1}{2}, -3\right) \text{ og } \left(\frac{5}{2}, -3\right)}$$

b) skjære x-aksen:

$$2 \sin(\pi x + \pi) - 1 = 0 \Rightarrow \sin(\pi x + \pi) = \frac{1}{2}$$

$$\Rightarrow \pi x + \pi = \frac{\pi}{6} + n \cdot 2\pi \Rightarrow x + 1 = \frac{1}{6} + 2n \Rightarrow x = -\frac{5}{6} + 2n$$

$$\underline{x = -\frac{5}{6} + 2 \cdot 0 = -\frac{5}{6}}, \underline{x = -\frac{5}{6} + 2 \cdot 1 = \frac{7}{6}}, \text{ ~~og } x = \frac{13}{6}~~$$

skjære y-aksen:

$$f(0) = 2 \cdot \sin(\pi \cdot 0 + \pi) - 1 = 2 \sin \pi - 1 = 2 \cdot 0 - 1 = \underline{-1}$$

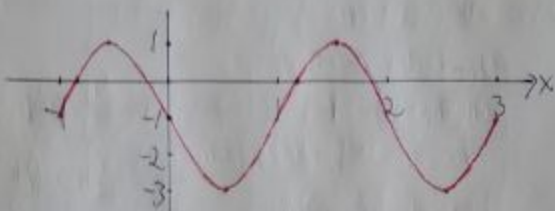
$$\Rightarrow \underline{y = -1}$$

→

$$f(-1) = 2 \sin(-\pi + \pi) - 1 = 2 \sin 0 - 1 = 0 - 1 = \underline{-1}$$

$$f(3) = 2 \sin(3\pi + \pi) - 1 = 2 \sin 4\pi - 1 = 2 \cdot 0 - 1 = \underline{-1}$$

Skizze & 1 grafen:



$$b) \quad 2 + \ln x + \frac{(\ln x)^2}{2} + \dots \quad k = \frac{\ln x}{2}$$

$$a) \quad -1 < k < 1$$

$$\underline{k > -1:}$$

$$\frac{\ln x}{2} > -1$$

$$\ln x > -2$$

$$e^{\ln x} > e^{-2}$$

$$x > e^{-2}$$

$$\underline{x > \frac{1}{e^2}}$$

$$\underline{k < 1:}$$

$$\frac{\ln x}{2} < 1$$

$$\ln x < 2$$

$$e^{\ln x} < e^2$$

$$\underline{x < e^2}$$

$$\Rightarrow \underline{x \in \left(\frac{1}{e^2}, e^2\right)}$$

$$b) \quad S = \frac{a_1}{1-k}$$

$$\frac{2}{1 - \frac{\ln x}{2}} = 4$$

$$1 - \frac{\ln x}{2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{\ln x}{2} = \frac{1}{2}$$

$$\ln x = 1$$

$$\underline{x = e} \quad (\text{gültig!})$$

$$A(-1, 3, 2) \quad B(2, 2, 1) \quad C(0, 1, 0) \quad T(5, 3, 8)$$

$$5) \quad a) \quad \underline{\underline{\vec{AB}}} = [2 - (-1), 2 - 3, 1 - 2] = \underline{\underline{[3, -1, -1]}}$$

$$\underline{\underline{\vec{AC}}} = [0 - (-1), 1 - 3, 0 - 2] = \underline{\underline{[1, -2, -2]}}$$

$$\underline{\underline{\vec{AB} \times \vec{AC}}} = [-1 \cdot (-2) - (-1) \cdot (-2), -(3 \cdot (-2) - (-1) \cdot 1), 3 \cdot (-2) - (-1 \cdot 1)]$$

$$= [2 - 2, -(-6 + 1), -6 + 1] = \underline{\underline{[0, 5, -5]}}$$

$$b) \quad \underline{\underline{V}} = \frac{1}{6} |(\vec{AB} \times \vec{AC}) \cdot \vec{AT}| = \frac{1}{6} |[0, 5, -5] \cdot [6, 0, 6]|$$

$$= \frac{1}{6} |0 \cdot 6 + 5 \cdot 0 - 5 \cdot 6| = \frac{1}{6} |-30| = \frac{1}{6} \cdot 30 = \underline{\underline{5}}$$

$$\underline{\underline{\vec{AT}}} = [6, 0, 6]$$

$$c) \quad \underline{\underline{n_\alpha}} = \frac{\vec{AB} \times \vec{AC}}{5} = \underline{\underline{[0, 1, -1]}}$$

geraden, plat $C(0, 1, 0)$

$$\Rightarrow \underline{\underline{\alpha}}: 0(x-0) + 1(y-1) - 1(z-0) < 0$$

$$y - 1 - z = 0$$

$$\underline{\underline{y - z = 1}}$$

$$7) 2x \cdot y' - 3y = 0$$

$$y' = \frac{3y}{2x}$$

$$A(2,2) \Rightarrow y' = \frac{3 \cdot 2}{2 \cdot 2} = \frac{3}{2} \text{ stemmer ikke}$$

$$B(-2,2) \Rightarrow y' = \frac{3 \cdot 2}{2 \cdot (-2)} = -\frac{3}{2} \text{ stemmer}$$

$$C(-2,-2) \Rightarrow y' = \frac{3 \cdot (-2)}{2 \cdot (-2)} = \frac{3}{2} \text{ stemmer}$$

$$D(2,-2) \Rightarrow y' = \frac{3 \cdot (-2)}{2 \cdot 2} = -\frac{3}{2} \text{ stemmer ikke}$$

$$9) a_1 = 2$$

$$a_n = a_{n-1} + 1$$

$$a_n = \frac{n^2 + n + 2}{2}$$

Sjekke for $n=1$:

$$a_1 = \frac{1^2 + 1 + 2}{2} = \frac{4}{2} = 2 \text{ stemmer!}$$

Sjekke for a_{n+1} :

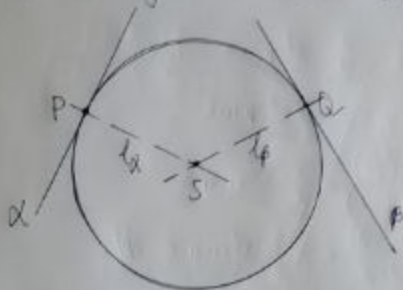
$$a_{n+1} = a_n + 1 + 1 = \frac{n^2 + n + 2 + 2(n+1)}{2}$$

$$= \frac{n^2 + n + 2 + 2n + 2}{2} = \frac{(n^2 + 2n + 1) + (n+1) + 2}{2}$$

$$= \frac{(n+1)^2 + (n+1) + 2}{2}$$

Danset bevis!

8) $\alpha: -2x + 2y - z = -2$ $P(-3, 7, -1)$ $\vec{n}_\alpha = [-2, 2, 1]$
 $\beta: -7x + 4y - 4z = 56$ $Q(-4, 5, -2)$ $\vec{n}_\beta = [-7, 4, -4]$



$$l_\alpha: \begin{cases} x = -3 - 2t \\ y = 7 + 2t \\ z = -1 - t \end{cases}$$

$$l_\beta: \begin{cases} x = -4 - 7s \\ y = 5 + 4s \\ z = -2 - 4s \end{cases}$$

$$l_\alpha = l_\beta$$

$$\begin{cases} -3 - 2t = -4 - 7s \\ 7 + 2t = 5 + 4s \end{cases}$$

$$-3 - 2t - 2t = -4 + 5 - 7s + 4s$$

$$4 = 1 - 3s$$

$$3 = -3s$$

$$s = -1$$

$$x = -4 - 7s = -4 - 7 \cdot (-1) = 3$$

$$y = 5 + 4s = 5 + 4 \cdot (-1) = 1$$

$$z = -2 - 4s = -2 - 4 \cdot (-1) = 2$$

$$\left. \begin{aligned} x &= 3 \\ y &= 1 \\ z &= 2 \end{aligned} \right\} S(3, 1, 2)$$

$$\vec{PS} = [3 - (-3), 1 - 7, 2 - (-1)]$$

$$= [6, -6, 3]$$

$$r = |\vec{PS}| = \sqrt{6^2 + (-6)^2 + 3^2} = \sqrt{81} = 9$$

$$\Rightarrow K: (x-3)^2 + (y-1)^2 + (z-2)^2 = 81$$