S2, vår 20, del 1

1) a) $f(x) = x^3 + 3e^x$

$\underline{f'(x)} = 3x^2 + 3e^x = \underline{\underline{3(x^2 + e^x)}}$

b) $g(x) = \dfrac{\ln(2x)}{x^2}$

$\underline{\underline{g'(x)}} = \dfrac{\cancel{\frac{1}{2x} \cdot 2} \cdot \cancel{x^2} - \ln(2x) \cdot 2x}{x^{\cancel{4}\,3}} = \underline{\underline{\dfrac{1 - 2\ln(2x)}{x^3}}}$

2) $\begin{bmatrix} 6x - y + 3z = 12 \\ 5x + 3y + z = 11 \\ 3x + 2y + z = 10 \end{bmatrix} \begin{matrix} \text{I} \\ \text{II} \\ \text{III} \end{matrix}$

$\underline{\text{II} - \text{III}}$:

$2x + y = 1 \qquad \text{IV}$

$\underline{\text{I} - 3\text{II}}$:

$-9x - 10y = -21 \quad \text{V}$

$\underline{\text{V} + 10\,\text{IV}}$:

$11x = -11$

$\underline{\underline{x = -1}}$

$\underline{\text{IV}}$

$y = 1 - 2x$

$\underline{y} = 1 - 2 \cdot (-1) = 1 + 2 = \underline{\underline{3}}$

$\underline{\text{III}}$

$z = 10 - 3x - 2y$

$\underline{z} = 10 - 3 \cdot (-1) - 2 \cdot 3$

$= 10 + 3 - 6 = \underline{\underline{7}}$

$\underline{\underline{x = -1,\ y = 3,\ z = 7}}$

3) $-8-3+2+7+\dots+987$

a) $\underline{d=5}$

$a_n = a_1 + (n-1)\cdot d$

$987 = -8 + (n-1)\cdot 5$

$987 = -8 + 5n - 5$

$5n = 987 + 8 + 5 = 1000$

$\underline{n} = \frac{1000}{5} = \underline{200}$

$S_n = \frac{n(a_1 + a_n)}{2}$

$\underline{\underline{S_{200}}} = \frac{\cancel{200}^{100}(-8+987)}{\cancel{2}}$

$= 100 \cdot 979 = \underline{\underline{97900}}$

b) $80 - 20 + 5 - \frac{5}{4} + \dots$

$\underline{k} = \frac{-20}{80} = \underline{-\frac{1}{4}}$

$S = \frac{a_1}{1-k}$

$\underline{\underline{S}} = \frac{80}{1-\left(-\frac{1}{4}\right)} = \frac{80}{1+\frac{1}{4}} = \frac{80}{\frac{5}{4}}$

$= \frac{\cancel{80}^{16}\cdot 4}{\cancel{5}} = \underline{\underline{64}}$

4)

$P(x) = x^3 - 9x^2 + 15x - 7$

a) $\underline{P(1)} = 1^3 - 9 \cdot 1^2 + 15 \cdot 1 - 7 = 1 - 9 + 15 - 7 = \underline{\underline{0}}$

$\Rightarrow$ $\underline{P(x)\text{ delelig med }(x-1)}$

b) $(x^3 - 9x^2 + 15x - 7) : (x-1) = x^2 - 8x + 7$

$$\begin{array}{l} \underline{x^3 - x^2} \\ \quad -8x^2 + 15x \\ \quad \underline{-8x^2 + 8x} \\ \qquad 7x - 7 \\ \qquad \underline{7x - 7} \\ \qquad\quad 0 \end{array}$$

$x^2 - 8x + 7 = 0$

$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \dfrac{8 \pm \sqrt{36}}{2}$

$x = \dfrac{8 \pm 6}{2}$ $\quad \underline{x} = \dfrac{14}{2} = \underline{7} \lor \underline{x} = \dfrac{2}{2} = \underline{1}$

$\Rightarrow P(x) = (x-1)(x-1)(x-7) \geq 0$

$\underline{P(x) \geq 0 \text{ for } x = 0 \lor x \in [7 \rightarrow\rangle}$

c) $\dfrac{x^2 - 2x + 1}{x^3 - 9x^2 + 15x - 7} = \dfrac{(x-1)^2}{(x-1)^2(x-7)} = \underline{\underline{\dfrac{1}{x-7}}}$

5) $f(x) = (x-1)^2(x-7)$ som på oppg 4!

a) Kan derfor derivere utifra $P(x)$ i oppg 4 og får:

$f'(x) = 3x^2 - 18x + 15 = 0 \quad |:3$

$x^2 - 6x + 5 = 0$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$= \dfrac{6 \pm 4}{2} \Rightarrow \underline{x = 5} \vee \underline{x = 1}$

$f''(x) = 6x - 18$

$\underline{f''(5) = 6 \cdot 5 - 18 = \underline{12}\ (\text{pos})} \Rightarrow$ Bunnpkt for $x = 5$

$f(5) = (5-1)^2(5-7) = 16 \cdot (-2) = \underline{-32}$

$\Rightarrow$ Bunnpkt i $(5, -32)$

Siden det er en 3. gradsfunksjon må det andre ekstremalpunktet være et topp-pkt.

$\Rightarrow f(1) = (1-1)^2 \cdot (1-7) = 0 \cdot (-6) = \underline{0}$

$\Rightarrow$ Topp-pkt i $(1, 0)$

b) $f(-1) = (-1-1)^2 \cdot (-1-7) = 4(-8) = \underline{-32}$ $\quad f(0) = \underline{-7}$

$\underline{f(x)}$:

c) $g(x) = -0{,}10 \cdot f(x) \quad D_g = [2, 6]$

a) Topp.pkt der f(x) hadde bunnpkt!

$\Rightarrow$ Vannstanden på sitt høyeste 5 dager etter at flommen startet.

$g(5) = -0{,}10 \cdot f(5) = -0{,}10 \cdot (-32) = 3{,}2$ meter

$\Rightarrow$ Vannstanden er da på 3,2 meter

d) Vendepkt for samme x-verdi som for f(x)

$f''(x) = 6x - 18 = 0 \Rightarrow \underline{x = 3}$

$\Rightarrow$ Vannstanden økte mest 3 dager etter at flommen startet

$\cancel{g'(3) = -0{,}10 \cdot f'(3) = -0{,}10 \cdot (3^2 - 6 \cdot 3 + 5) = -0{,}10 (9 - 18 + 5)}$

$\cancel{= -0{,}10 \cdot (-4) = 0{,}4}$

~~$\Rightarrow$ Den økte da med 0,4 meter / dag~~

$f'(x) = 3x^2 - 18x + 15 = 3(x^2 - 6x + 5)$

$g'(3) = -0{,}10 \cdot f'(3) = -0{,}10 \cdot 3(3^2 - 6 \cdot 3 + 5)$

$= -0{,}30 \cdot (9 - 18 + 5) = -0{,}30 \cdot (-4) = \underline{1{,}2}$

Vannstanden økte da med 1,2 meter / døgn

6) a) $\underline{K'(100)} = \frac{(1200-700)kr}{100\ enh} = \frac{500\ kr}{100\ enh} = \underline{5\ kr}$

$\underline{\bar{E}(100)} = \frac{K(100)}{100} = \frac{1200\ kr}{100} = \underline{12\ kroner}$

b) $\bar{E}(x) = \frac{K(x)}{x} \qquad K(x) = x \cdot \bar{E}(x)$

$$\underline{\bar{E}'(x)} = \frac{K'(x)\cdot x - K(x)\cdot 1}{x^2} = \frac{K'(x)\cdot x - \bar{E}(x)\cdot x}{x^2}$$

$$= \frac{\cancel{x}(K'(x) - \bar{E}(x))}{\cancel{x}\cdot x} = \underline{\frac{K'(x) - \bar{E}(x)}{x}}$$

c) $\underline{\bar{E}'(100)} = \frac{K'(100) - \bar{E}(100)}{100} = \frac{(5-12)kr}{100}$

$$= \frac{-7kr}{100} = \underline{-0{,}07\ kr}\ \underline{(-7\ øre)}$$

<u>Dette forteller oss at enhetskostnaden går ned med 0,07 kroner (7 øre) når vi øker produksjonen fra 100 til 101 enheter</u>

7) a) $P(G) = 0{,}20 \Rightarrow p = 0{,}20$

$n = 100$ X = ant gule drops

$\underline{E(X)} = n \cdot p = 100 \cdot 0{,}20 = \underline{20}$

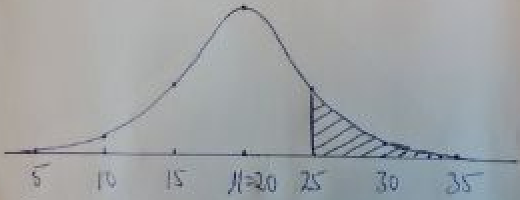
$\underline{Var(X)} = n \cdot p \cdot (1-p) = 100 \cdot 0{,}20 \cdot 0{,}80 = 100 \cdot 0{,}16 = \underline{16}$

b) $\underline{\sigma} = \sqrt{Var(X)} = \sqrt{16} = \underline{4}$

$\underline{P(X \geq 25)} = 1 - P(X \leq 25) = 1 - P\left(Z \leq \frac{x-\mu}{\sigma}\right)$

$= 1 - P\left(Z \leq \frac{25-20}{4}\right) = 1 - P(Z \leq 1{,}25) = 1 - 0{,}8944 = \underline{0{,}1056}$

c)



d) $P(20-a \leq X \leq 20+a) = 0{,}90$

Likevekt på begge sider av $X \Rightarrow \underline{P(X \leq 20-a)}$

$= \frac{1-0{,}90}{2} = \frac{0{,}10}{2} = \underline{0{,}05}$

$\Rightarrow P\left(Z \leq \frac{(20-a)-20}{4}\right) = P\left(Z \leq \frac{-a}{4}\right) = 0{,}05$

$= \frac{-a}{4} = -1{,}645 \Rightarrow \underline{a} = 4 \cdot 1{,}645 = \underline{6{,}58} \approx \underline{6{,}6}$

Forteller at 90% av posene har mellom 13,4 og 26,6 gule drops