Løsningsforslag R2 H22

1. a) $f(x) = x^3 + \sin(\pi x)$

$f'(x) = 3x^2 + \pi\cos(\pi x)$

b) $g(x) = \ln(2+\cos x)$

$g'(x) = -\sin x \cdot \frac{1}{2+\cos x} = \underline{\underline{\frac{-\sin x}{2+\cos x}}}$

2. a) $\int \left(\cos\left(\frac{x}{2}\right) - e^{-x} + 1\right) dx =$

$= \underline{\underline{2\sin\left(\frac{x}{2}\right) + e^{-x} + x + C}}$

b) $\int_0^{\sqrt{\pi}} x \cdot \sin x^2 \, dx = \int_0^{\sqrt{\pi}} x \cdot \sin u \, \frac{du}{2x} =$

$= -\frac{1}{2}\left[\cos x^2\right]_0^{\sqrt{\pi}} = -\frac{1}{2}\left(\cos\pi - \cos 0\right) =$

$= \underline{1}$

3. a)

$$y' = \frac{2x^2}{y} \qquad y(0) = 3$$

$$\int y\,dy = 2\int x^2\,dx \qquad 3 = \sqrt{c}$$

$$\frac{1}{2}y^2 + c = \frac{2}{3}x^3 + c \qquad \underline{c = 9}$$

$$y^2 = \frac{4}{3}x^3 + c \qquad \underline{\underline{y = \sqrt{\frac{4}{3}x^3 + 9}}}$$

$$y = \pm\sqrt{\frac{4}{3}x^3 + c}$$

b)

$$y'' + 9y = 9x \qquad y = 2\sin(3x) + x$$

$$-18\sin(3x) - 18\sin(3x) + 9x = 9x \qquad y' = 6\cos(3x) + 1$$

$$\underline{\underline{9x = 9x}} \qquad y'' = -18\sin(3x)$$

Dvs. da må $y = 2\sin(3x) + x$ være en løsning på differensiallikningen

4.

a) $a_3 = 8 \qquad a_{23} = 68$

$a_n = a_1 + (n-1)d$

$\text{I} \quad 8 = a_1 + 2d \quad \Rightarrow \quad a_1 = 8 - 2d$

$\text{II} \quad 68 = a_1 + 22d \qquad a_1 = 8 - 2d = 8 - 6 = 2$

$\text{II} \quad 68 = 8 - 2d + 22d \qquad a_n = 2 + (n-1)3 =$

$60 = 20d \qquad = 2 + 3n - 3 =$

$\underline{d = 3} \qquad = \underline{3n - 1}$

$a_{100} = 3 \cdot 100 - 1 = \underline{299}$

$S_{100} = \frac{(a_1 + a_{100})}{2} \cdot 100 = 301 \cdot 50 = \underline{15050}$

b) $a_2 = a_1 + 4 = a_1 + d \Rightarrow \underline{d = 4}$

$a_n = a_1 + (n-1)4 = \underline{a_1 + 4n - 4} \qquad a_{10} = a_1 + 36$

$S_{10} = \frac{a_1 + (a_1 + 36)}{2} \cdot 10 = \underline{(a_1 + 18) \cdot 10}$

$(a_1 + 18) \cdot 10 = 240 \qquad a_n = 6 + 4n - 4 = \underline{4n + 2}$

$\underline{a_1 = 6} \qquad a_{20} = 4 \cdot 20 + 2 = \underline{82}$

$S_{20} = \frac{6 + 82}{2} \cdot 20 = \underline{880}$

5. a) $f(x) = A\sin(cx + \varphi) + d$

$A = \frac{3-(-5)}{2} = \underline{4}$ $\qquad P = \frac{11\pi}{6} - \left(-\frac{\pi}{6}\right) = \underline{2\pi}$

$d = \frac{3-5}{2} = \underline{-1}$ $\qquad c = \frac{2\pi}{P} = \underline{1}$

$\varphi = -cx_0 = -1 \cdot \frac{\pi}{3} = \underline{-\frac{\pi}{3}}$

$$\underline{\underline{f(x) = 4\sin\left(x - \frac{\pi}{3}\right) - 1}}$$

b) $\sin x - \sqrt{3}\cos x = 0 \qquad x \in [0, 3\pi]$

$\tan\varphi = \frac{\sqrt{3}}{1} = \underline{\sqrt{3}} \Rightarrow (1, -\sqrt{3}) \Rightarrow \underline{\varphi = -\frac{\pi}{3}}$

$A = \sqrt{1+3} = \underline{2}$

$2\sin\left(x - \frac{\pi}{3}\right) = 0$

$\sin\left(x - \frac{\pi}{3}\right) = 0$

$x = \frac{\pi}{3} + k\pi$

$x = \frac{\pi}{3} \lor x = \frac{4\pi}{3} \lor x = \frac{7\pi}{3}$

$$\underline{\underline{L\left\{\frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}\right\}}}$$

6. a)

$$x^2+y^2+z^2-4x+2y-6z=11$$

$$(x-2)^2+(y+1)^2+(z-3)^2=11+4+1+9$$

$$\underline{(x-2)^2+(y+1)^2+(z-3)^2=25}$$

Sentrum er da $(2,-1,3)$ og
radius er $r=\sqrt{25}=5$

b)

$S=(2,-1,3)$
$P=(6,-4,3)$

$\vec{SP}=[4,-3,0]$

$\vec{n_\alpha}=\vec{SP}=[4,-3,0]$

$$\alpha: 4(x-6)+(-3)(y+4)+0(z-3)=0$$

$$\underline{\alpha: 4x-3y=36}$$

c)

$|\vec{SP}|=|\vec{SQ}|\ \underline{=5}$ $\qquad Q=(4t+2,-3t-1,3)$

$$|\vec{SQ}|=\sqrt{16t^2+9t^2}=\sqrt{25t^2}$$

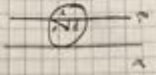
$\sqrt{25t^2}=5$ $\qquad t=1$ gir P

$5t=\pm5$ $\qquad$ da gir $t=-1$ Q

$\underline{t=\pm1}$

$$\underline{Q=(-2,2,3)}$$

6 d)



$d = $ [illegible]

$r^2 = d^2 + R^2$

$R = \sqrt{25-4} = \sqrt{21}$

Radius til skjæringssirkelen = $R = \sqrt{21}$

7. a)

Oline bruker det bestemte integralet og må sette inn a først og 1 sist.
Det ubestemte integralet kan da være
$x^3 - \ln x + 3x + C$

$\int_1^a f(x)\,dx = [x^3 - \ln x + 3x]_1^a = a^3 - \ln a + 3a - (1 - 0 + 3) =$

$= \underline{a^3 - \ln a + 3a - 4}$

Svaret til Oline kan altså ikke være riktig.

b) $f(x) = (x^3 - \ln x + 3x)' = 3x^2 - \frac{1}{x} + 3$

8.

a) $a = \frac{\Delta y}{\Delta x} = \frac{3}{2}$ $\qquad$ $y = \frac{3}{2}x$

$$\pi \int \frac{9}{4}x^2\,dx = \frac{3\pi}{4}x^3$$

$$\frac{3\pi}{4}\left[x^3\right]_0^h = \frac{3\pi}{4}h^3$$

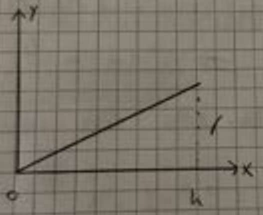
$$\frac{3\pi}{4}h^3 = 48\pi$$

$$h^3 = 64$$

$$h = 4$$

h må være 4 for at volumet skal være 48π

b)



$a = \frac{\Delta y}{\Delta x} = \frac{r}{h}x$

$$V = \pi \int_0^h \left(\frac{r}{h}x\right)^2 dx = \frac{\pi r^2}{h^2}\int_0^h x^2\,dx =$$

$$= \frac{\pi r^2}{3h^2} \cdot h^3 = \frac{\pi r^2 h}{3} = \frac{1}{3}\pi r^2 h$$

Har vist at volumet er gitt ved

$V = \frac{1}{3}\pi r^2 h$.